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The Fractal Dimension Theory of Ismail's Third Entropy with Fractal Applications to CubeSat Technologies and Education

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Abstract

As a mathematical concept, entropy is the first and most advanced of its kind. Only a handful of studies have examined the fractal dimensions of fundamental entropies, including Shannon, Tsallis, and Rényi entropic expressions, in the literature. This served as a deliberative source of inspiration for further investigation into the direction of a cohesive information-theoretic fractal theory. The current study begins with introducing my third entropy formula, which is Ismail's entropy, or ($H_{III}^{(\gamma,q,a,b)}$) as a novel generalisation to Shannonian entropy with a forward-thinking connection to both long- and short-range interactions, or SRIs and LRIs, respectively. This would obviously lead to a fourfold unification with modern physics and statistical mechanics. This gives the study greater validity and weight. This study determines the fractal dimension of $H_{III}^{(\gamma,q,a,b)}$. This study played a significant role in drawing attention to the value of fractal geometry for the space industry and education, both of which are vital for advancing our civilizations. Consequently, a few possible fractal applications to education and CubeSat technologies are emphasized. Thus, the extensive hunt for additional ground-breaking research has concluded. Of course not this served as another motivation for me to suggest fresh, open challenges that would allow the scientific community to explore more avenues for research. Lastly, a combined overview is given with thought-provoking research topics and the next study stage.

Keywords: Fractal dimension (D_f), Ismail's third entropy ($H_{III}^{(\gamma,q,a,b)}$), CubeSat technologies, Education.

1 | Introduction

Shannon's entropy reads [1–3]:

$$H(X) = \sum_i p(x_i) I(x_i) = -\sum_i p(x_i) \ln(p(x_i)). \quad (1)$$

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The probability of the i^{th} event is given by the equation $p(x_i)$. More fundamentally, Ismail's third entropy, namely, $H_{III}^{(\gamma,q,a,b)}$ reads as

$$H_{III}^{(\gamma,q,a,b)}(x) = \frac{c}{\gamma(1-q)(a-b)}\left[(x)^{a\gamma} - (x)^{b\gamma}\right]. \tag{2}$$

where c is a positive constant, $x = \sum_{n=0}^{\infty}(p(n))^q$, $1 > q > 0.5, a > 0, b \in \mathbb{R}$ or $b > 0, a \in \mathbb{R}$ with $a \neq b, q, \gamma \in \mathbb{R}, x = \sum_n p_{q,Z}(n)^q$.

Because of the statistical evaluation of spatial dimension complexity, entropy and D_f are connected. A pattern's ability to occupy space is measured by fractals, which get their name from their distinctive scaling tendency. Information theory and pattern geometry interact intricately, and this link clarifies these relationships [4–7]. Lewis Fry Richardson looked at how the length of the stiff stick used to measure coastlines could affect the length of the coastline measured in [8]. *Fig. 1* [9] shows Mandelbrot's reference to earlier research in [8–10].

There are several formal mathematical formulations of D_f that can be studied. In one such formulation, the scaling factor (ε), D_f , and the number of sticks (N) needed to cover a shoreline are related through formulae. These equations aid in estimating the fractal patterns' scaling characteristics and complexity in spatial dimensions:

$$N \propto \varepsilon^{-D_f}. \tag{3}$$

$$\ln N = -D_f = \frac{\ln N}{\ln \varepsilon}. \tag{4}$$

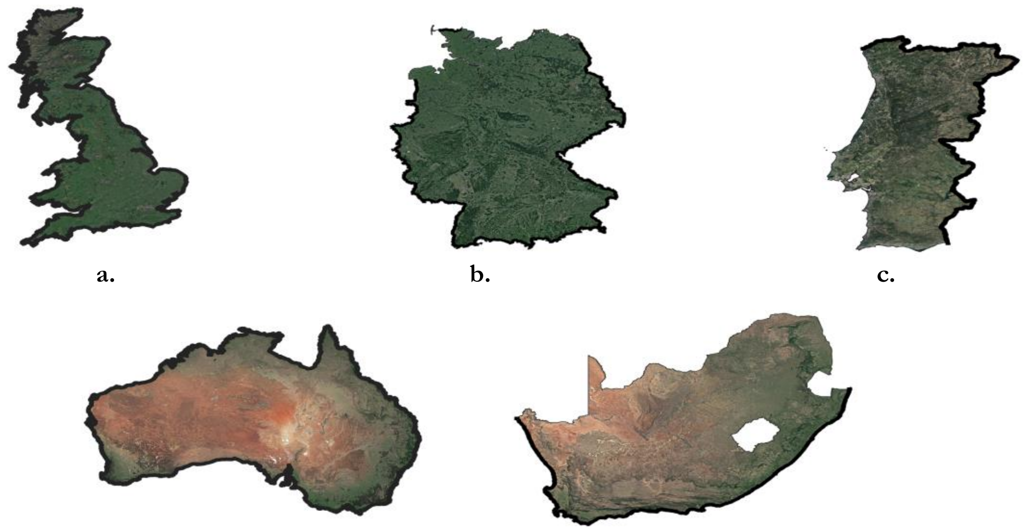


Fig. 1. Mandelbrot's reference to earlier research in [8–10]; a. coastline of Great Britain, b. land frontier of Germany, c. land frontier of Portugal, d. coastline/border of Australia, e. coastline of South Africa.

An area of several Coastal lines were captured via Google Earth satellite imagery, demonstrating the process of painting portraits.

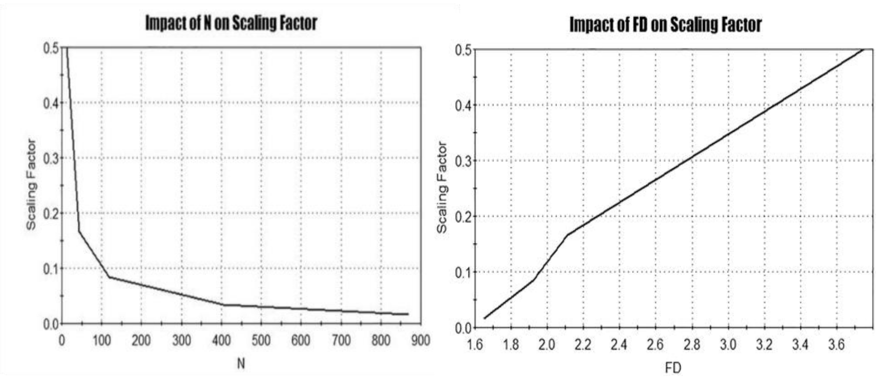


Fig. 2. A two-dimensioanl visualisations of rigid sticks for the GIMP program.

This paper flowchart reads: 1) Introduction, 2) Methodology, 3) Results and discussion, 4) D_f CubeSat technologies and education, 5) Conclusion.

2 | Methodology

Undoubtedly, fractal dimension theory is the first of its own kind in mathematics, based on its influential applicability to numerous fields of human knowledge. Having introduced $H_{III}^{(\gamma,q,a,b)}$, fractal dimension is applied to $H_{III}^{(\gamma,q,a,b)}$ to uncover the dominancy of the derivations, which reduce to the corresponding subcases in information theory. Therefore, a new unified information-theoretic fractal theory is developed. More importantly, to add a more application-based approach, it was necessary to highlight some fractal geometric applications to two important fields of knowledge in our lives, namely CubSats, and education. Clearly, this will open many research thoughts on the importance of fractal geometry in revolutionizing human knowledge.

3 | Results and Discussion

In [9], an exposition of D_f for Shannon entropy [1], Rényi entropy [3], [11–13], and Tsallis entropy [13] was undertaken for events with equal probabilities, i.e., $p(i) = \frac{1}{N}$.

D_f is defined by the Shannonian entropic [9] concept as

$$D_{f,s} = \lim_{\varepsilon\to 0}\frac{\ln N}{\ln\frac{1}{\varepsilon}}. \tag{5}$$

Following the same path, Rényi dimension [12] reads

$$D_{f,R} = \lim_{\varepsilon\to 0}\frac{\ln N}{\ln\frac{1}{\varepsilon}}. \tag{6}$$

It is to be noted that $H_{I}^{(\gamma,q,a,b)}$ generates several major entropies in the literature.

I. The $H_{III}^{(1,q,a,b)}$ is the $Z_{a,b}$ entropy [4].

II. The $H_{III}^{(1,q,1,0)}$ is the Tsallis entropy.

III. The $H_{III}^{(1,q,k,-k)}$ is the k- entropy.

IV. The $H_{III}^{(1,q,a,0)}$ is the Sharma–Mittal entropy [4].

V. $H_{III}^{(1,1,0,1)}$ is the Shannonian entropy.

Theorem 1. For $H_{I}^{(\gamma,q,a,b)}$ *Eq. (2)*, $D_f(H_{III}^{(\gamma,q,a,b)})$ is given by

$$D_f\left(H_{III}^{(\gamma,q,a,b)}\right) = \lim_{\epsilon\to 0}\frac{\frac{1}{(1-q)(a-b)\gamma}\left(N^{(1-q)a\gamma} - N^{(1-q)b\gamma}\right)}{\ln\frac{1}{\epsilon}}. \quad (7)$$

Proof:

$$D_f\left(H_{III}^{(\gamma,q,a,b)}\right) = \frac{1}{\gamma(1-q)(a-b)}\lim_{\epsilon\to 0}\frac{\left(\sum_{i=1}^{N}\left(\frac{1}{N}\right)^q\right)^{a\gamma} - \left(\sum_{i=1}^{N}\left(\frac{1}{N}\right)^q\right)^{b\gamma}}{\ln\frac{1}{\epsilon}}$$

$$= \frac{1}{\gamma(1-q)(a-b)}\lim_{\epsilon\to 0}\frac{\left(\frac{1}{N}\right)^{aq\gamma}\left(\sum_{i-1}^{N} 1\right)^a - \left(\frac{1}{N}\right)^{bq\gamma}\left(\sum_{i-1}^{N} 1\right)^b}{\ln\frac{1}{\epsilon}} =$$

$$= \frac{1}{\gamma(1-q)(a-b)}\lim_{\epsilon\to 0}\frac{\left(\frac{1}{N}\right)^{aq\gamma}(N)^a - \left(\frac{1}{N}\right)^{bq\gamma}(N)^b}{\ln\frac{1}{\epsilon}}$$, as claimed. (*Eq. (7)*).

Corollary 2. $D_f\left(H_{III}^{(\gamma,q,a,b)}\right)$ *Eq. (7)* satisfies:

I. $\lim_{a\to 1, b\to 0, \gamma\to 1} D_f\left(H_{III}^{(\gamma,q,a,b)}\right) = \lim_{\epsilon\to 0}\frac{\frac{1}{(1-q)}\left(N^{(1-q)a} - 1\right)}{\ln\frac{1}{\epsilon}} = D_{f,T}$, T = Tsallis.

II. $\lim_{a\to k. b\to -k. \gamma\to 1} D_f\left(H_{III}^{(\gamma.q.a.b)}\right) = D_{f.K}$. K = Kaniadakis.

III. $\lim_{\gamma\to 0} D_f\left(H_{III}^{(\gamma.q.a.b)}\right) = \lim_{\gamma\to -1. a.b.q\to 0} D_f\left(H_{III}^{(\gamma.q.a.b)}\right) = \lim_{\gamma.q\to -1. a.b\to 0} D_f\left(H_{III}^{(\gamma.q.a.b)}\right) = D_{f.s} = D_{f.R}$,

IV. $\lim_{\gamma\to 1} D_f\left(H_{III}^{(\gamma,q,a,b)}\right) = D_{Z_{a,b}}$.

4 | D_f Applications to Cubesat Technologies and Education

4.1 | D_f Applications to CubSat Technologies

A UHF band square Koch fractal slot antenna's design, production, and testing are described in [14]. With a length of 56.56 centimetres, the antenna is designed to fit inside the typical CubeSat faces, which are 10 cm by 10 cm. A schematic illustration of the suggested antenna design is shown in *Fig. 3* [14], where it is positioned on the surface of a 1U CubeSat, which measures 10 cm by 10 cm. The antenna's diagonal length, which represents the antenna's overall length at each iteration, is 14.14 cm for the 0th iteration, 28.28 cm for the first iteration, and 56.56 cm for the second.

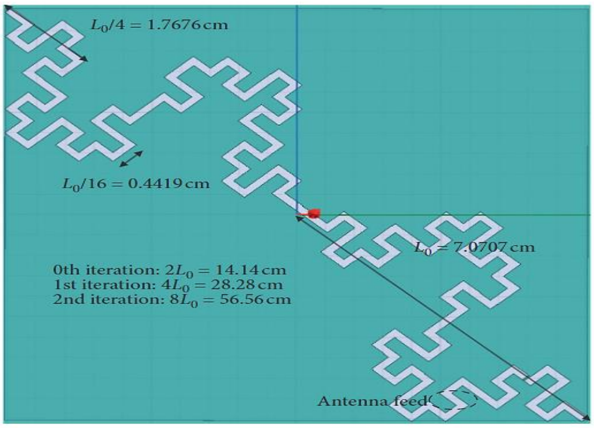


Fig. 3. A schematic of a 1U CubeSat satellite with a fractal slot antenna mounted on one face.

The antenna geometry consists of multiple iterations of koch curves, with each iteration increasing the total length of the antenna. The dimensions and placement of the antenna feed are determined based on parametric studies and simulations to achieve better impedance matching.

Fig. 4 [14] depicts a top view of the slot antenna, which allows for a clear visualization of the antenna's physical structure and layout

Fig. 4. The top view of the implemented antenna, which has dimensions of 20.4 cm×25.4 cm.

This antenna is designed for use in a CubeSat satellite and is part of a fractal slot antenna system. The top view shows the central part of the antenna, which measures 10 cm×10 cm, located on a Printed Circuit Board (PCB) laminate.

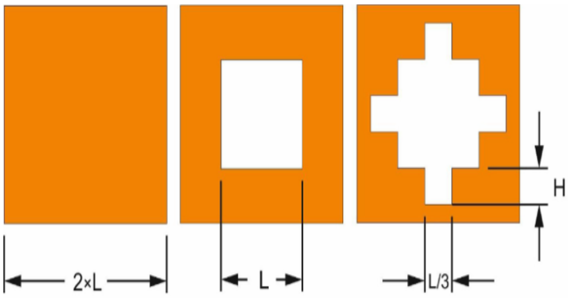


Fig. 5. An indication of the process of designing and constructing the specific component of the antenna responsible for emitting electromagnetic waves.

The details regarding the specific methodology or techniques used for this formation are not provided in the given context [14].

The antenna main characteristics' simulations are visualized in *Fig. 6* and *Fig. 7* [15], show the reflection coefficient, impedance bandwidth, antenna patterns, realized gain, and efficiency at different frequencies

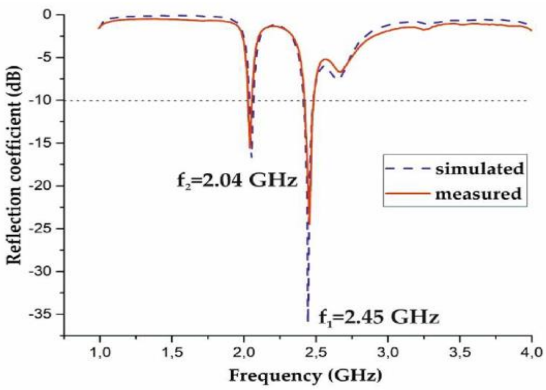
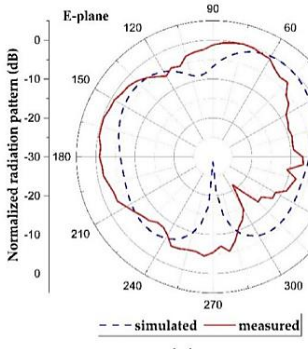
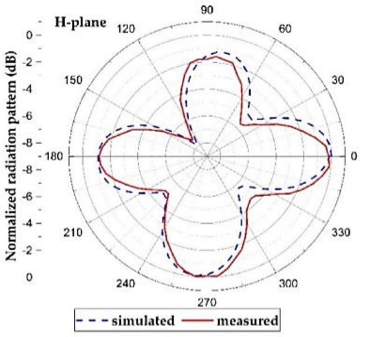


Fig. 6. The authors [15] present these results in a graph where the dashed line represents the modeled reflection coefficient, and the full line represents the measured reflection coefficient.



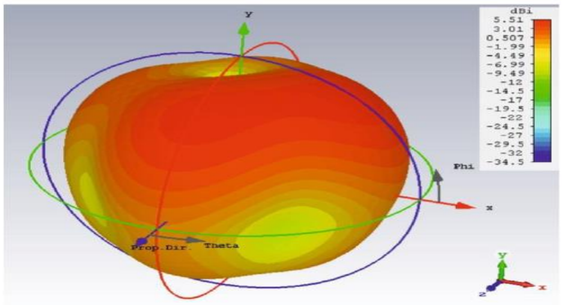
a.



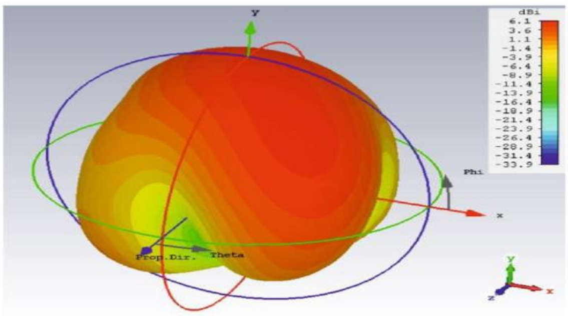
b.

Fig. 7. The simulation results represented by dashed lines, and the measurement results by full lines [15].

Fig. 8 [15] displays the three-dimensional radiation pattern of an antenna, which provides information about how the antenna radiates electromagnetic energy in different directions at these specific frequencies.



a.



b.

Fig. 8. The simulation results and measurements are presented, indicating the antenna's directional characteristics at these frequencies.

A specialised fractal antenna, a Field Programmable Gate Array (FPGA), and Software-Defined Radio (SDR) are part of the BuhoSat's main payload [16] that is designed to detect synchrotron emission from electrons in the Van Allen belt. Scientists can learn important details about the characteristics of the ionised gas in the F region of the ionosphere and the Earth's atmosphere by examining the spectra recorded by this device.

On another separate note, the main instrument of the BuhoSat is designed to capture electromagnetic radiation centered at 3.8 GHz [16]. It utilizes a fractal antenna based on the Sierpinski carpet (corresponding to $D_f, N = 3$ and $\varepsilon = 0.5$), which allows it to capture synchrotron radiation emitted by spinning electrons about magnetic lines. A powerful FPGA and SDR are used to process the captured radiation, enabling the spectrum analysis in the frequency domain. This is illustrated by *Fig. 9* [16].

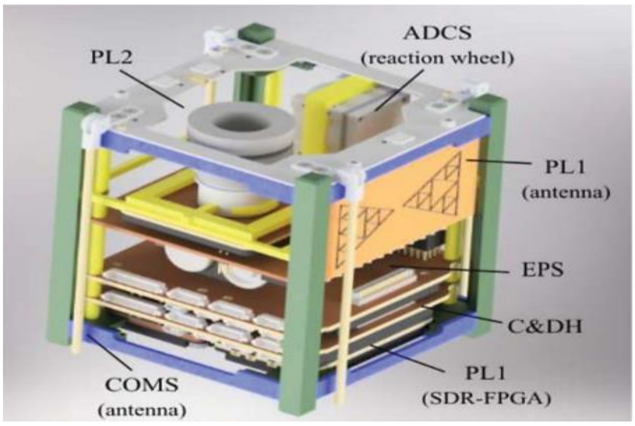


Fig. 9. The text refers to a graphical representation of BuhoSat, which is a visual depiction of the structure and mechanisms of the satellite.

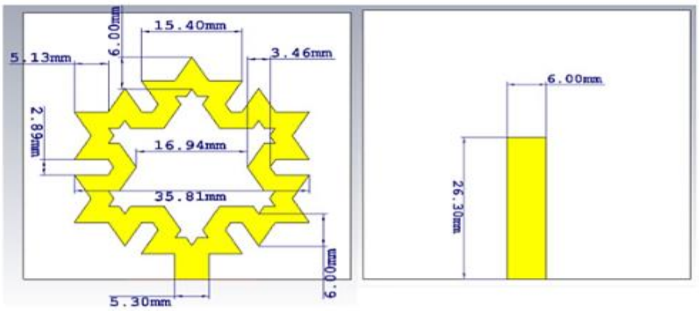
The BuhoSat mission requirements provide specifications and verification criteria for various aspects of the satellite's design, such as protection of components, power supply, communication capabilities, imaging capabilities, data storage, and environmental requirements. The graphical representation serves as a visual aid to understand the overall design and functionality of BuhoSat.

Small and lightweight, CubeSats (known as LEO satellites) can be constructed with off-the-shelf parts and designed to operate in Low Earth Orbit (LEO). Compared to medium-sized satellites, they are less functional but can be developed more quickly and at a lower cost. CubeSats depend on the Telemetry, Tracking, and Communications (TTC) subsystem, and creating antennas for them is difficult because of their limited size, weight, and power [17]. *Fig. 10* [17] lists the three typical CubeSat types 1U, 2U, and 3U and their corresponding sizes.

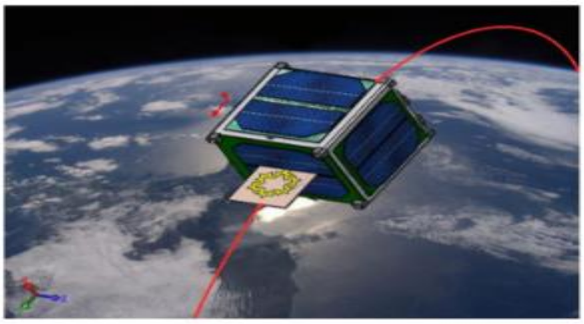
a. **b.** **c.**

Fig. 10. Different sizes of CubeSat models, specifically 1U, 2U, and 3U.

To address this issue, a deployable microstrip patch antenna with a fractal architecture for 1U CubeSats was created [17]. By utilising the Koch snowflake fractal structure (corresponding to $D_f, N = 4$ and $\varepsilon = \frac{1}{3}$). Its omnidirectional layout does, however, result in less gain. The proposed antenna (see *Fig. 11* [17]) achieves a compact size, high gain, minimal reflection coefficient, big bandwidth, and good impedance matching.



a.



b.

Fig. 11. The proposed antenna.

4.2 | D_f Applications to Education

In [18], fractal pedagogy's theoretical and methodological foundations, as well as its principles and features, were discussed. The research aimed [18] to identify the components of personal and professional self-development for teachers, including motivation, design, practical activities, reflection, and emotional volition. So, in principle, [18] provided a new perspective on the problem of teacher self-development and suggests that fractal methodology can serve as a basis for further psychological and pedagogical research in this area.

The principle of fractality [18] applies to various social processes, allowing us to understand different aspects of life from a fractal perspective. Examples of fractal organization can be seen in socio-cultural systems such as language, music, architecture, and patterns, highlighting the multifaceted nature of these phenomena. The term "conceptual fractal" [18] describes the self-multiplying nature of socio-cultural practices and their organization at different levels and scales, ultimately contributing to the overall structure of the socio-cultural world.

More fundamentally, Fractals [18] are not a new concept in technical sciences and are used as a basis for various scientific disciplines. Fractal methodology involves studying social systems at different levels, recognizing cyclical trends and the legitimacy of events, and creating socio-political and other fractal models [18]. In the context of personal and professional self-development in education, fractal methodology is employed to understand the process from a fractal theory perspective, with an integrated-ecological methodological approach being the leading approach in studying the organization of this process [18].

The importance of highly qualified specialists in ongoing reforms and developing a mature training system in Uzbekistan [19] was highlighted. Fractals, complex objects based on simple laws, have similarities with natural phenomena and can be used to predict various factors, such as the growth rate of plant root systems and the mass of straw dependent on branch length [19].

According to [19], it is recommended to incorporate fine arts and teaching practices that consider the specificities of fine and applied arts to enhance students creative abilities. By utilizing facts in applied art compositions, color palettes, and graphics, students can develop their observation skills, stimulate their imagination, and recognize the fractal nature of objects [19]. Fractals, complex structures governed by simple laws, share similarities with various natural phenomena, ranging from crystals and ecosystems to biological objects, highlighting the interconnectedness of art and nature [19].

A novel fractal approach to teaching physical and mathematical disciplines is presented to promote independent and original computer modelling of natural processes [20]. Encapsulation, inheritance, and polymorphism, three core tenets of object-oriented programming, have demonstrated their power in influencing how various academic fields perceive information architecture's physical and mathematical components [20]. This method can potentially be applied to other areas of physics, as shown [20]. The study of the Geometric Optics and Wave Optics portions of physics is used to illustrate how the generated iterations of the fractal structure are displayed [20], including a new iteration results in a high-quality and in-depth perception of new information, demonstrating how each iteration is characterised by synergy [20].

A fractal is an information-branched structure whose spatial dimension is characterised by a fractional number in the context of applying physical and mathematical education [20]. When creating complex structures, this property of fractality ensures the accuracy and completeness of information flows. However, minimal energy dissipation and energy-informational self-similarity are used to achieve complexity [20]. This demonstrates the effectiveness and appropriateness of their implementation in teaching science pedagogical subjects, which are currently underdeveloped and underargued. Geometric, algebraic, and stochastic fractal characteristics (*Fig. 12*, [20]) can be seen as characteristics of how the instructional process works.

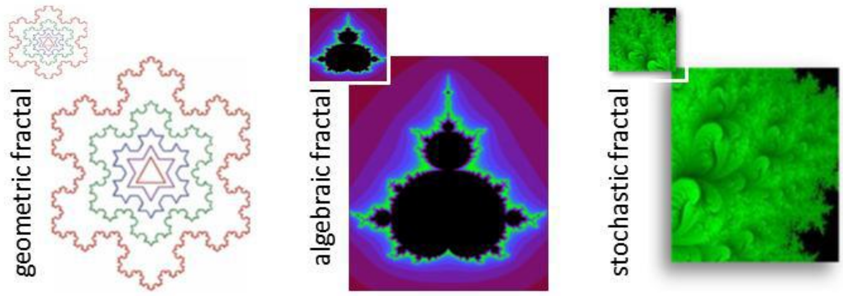


Fig. 12. Koch's snowflake example of a geometric fractal, the Mandelbrot set illustration of an algebraic fractal, and the bubble flow illustration of a stochastic fractal.

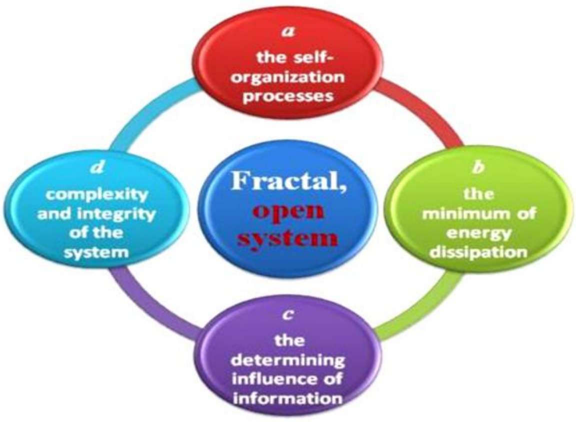


Fig. 13. The fractal open information system [20]

A complicated application of aspects of geometric, algebraic, and stochastic fractals was used to build iterations of the fractal structure as it relates to the study of the physics sections Geometric Optics and Wave Optics [20]. In *Figs. 14-17*, the concepts and growth of the fractal structure's branches are depicted. It should be emphasised that the encapsulation, inheritance, and polymorphism concepts used in computer modelling are also founded on these concepts. It is object-oriented modelling, in other words.

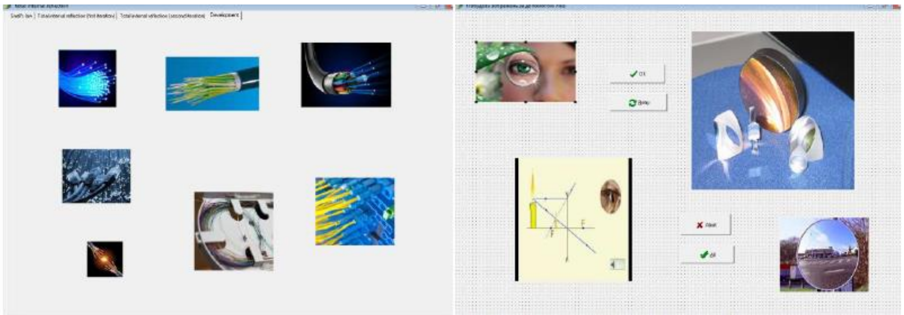


Fig. 14. Computer modelling of Snell's laws is an illustration of a branch fractal structure [20].

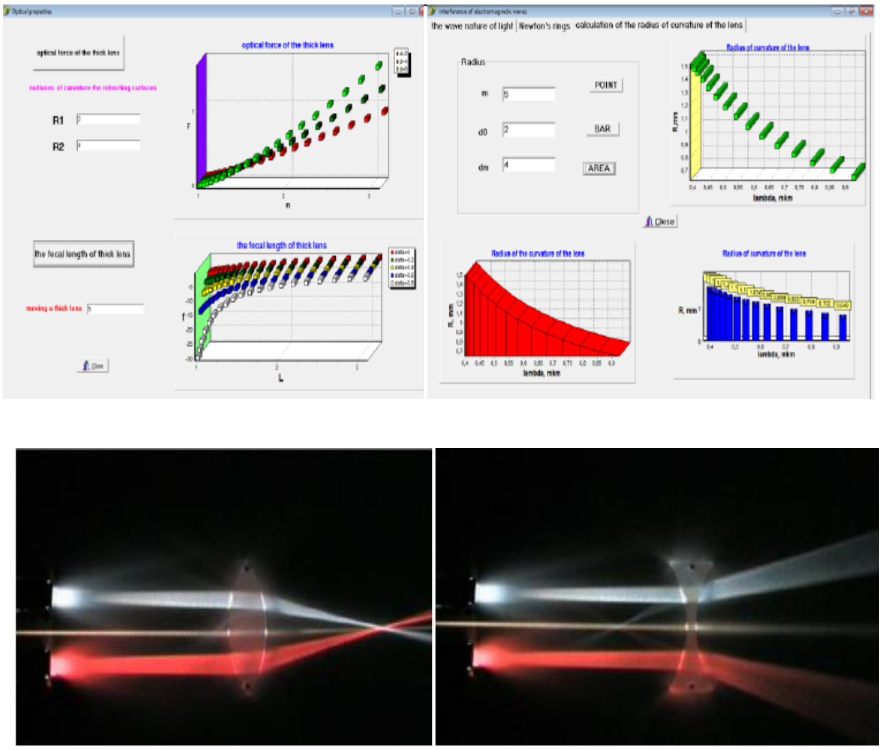


Fig. 15. The iteration process and visual interface for modelling optical parameters [20]

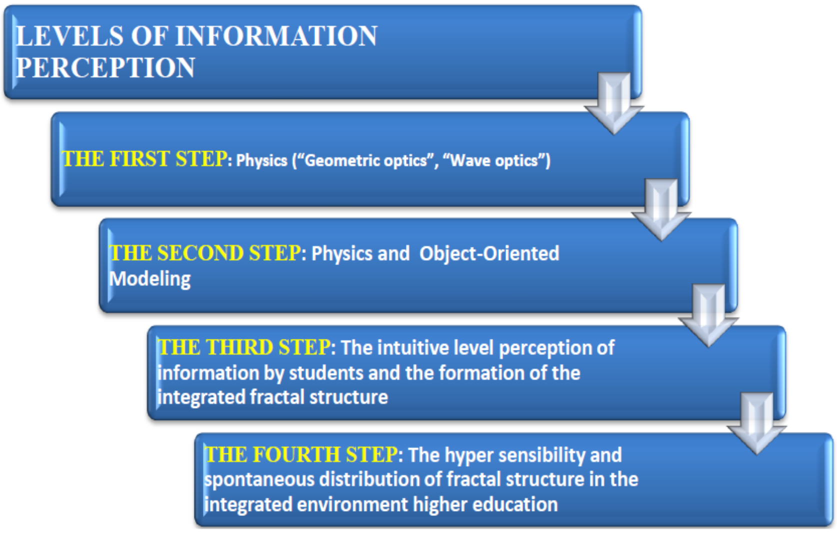


Fig. 16. Procedures and consequences for creating a fractal structure on the level of perception of physical education data [20].

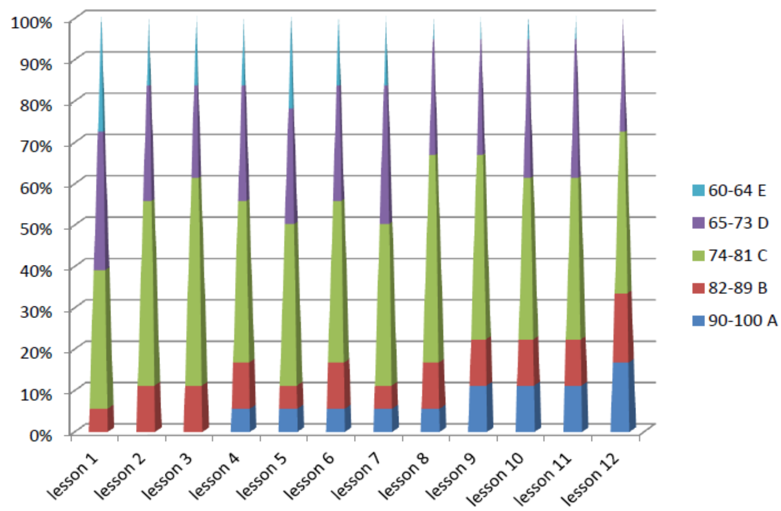


Fig. 17. The outcome of an experiment conducted during the 2018–2019 academic year in the study area "methods of teaching physics," which included computer modelling of the physics sections "geometrical optics" and "wave optics" [20].

5 | Closing Remarks With the Next Phase of Research

The theory $D_f\,(H_{III}^{(\gamma,q,a,b)})$ is established in this elaboration. More importantly, this work has shown how fractals can help various multidisciplinary sciences progress. These are some suggested unsolved issues:

I. Can the mathematical issue of calculating the threshold formalism of the derived fractal dimension of $H_{III}^{(\gamma,q,a,b)}$ be solved regarding each of the four parameters, q, γ, a, b and N involved?

II. As we get closer to the $H_{III}^{(\gamma,q,a,b)}$'s Snow Kochflake fractal and the Sierpiniski Gasket dimensions, is it possible to solve the mystery of randomness, thermodynamics, fractal geometry, and statistical physics by unlocking the threshold of both fractal dimensions, namely for the long-range interaction descriptor, q?

III. According to [17], the main flaw of reconfigurable reflect arrays is their narrow gain bandwidth, which can be as low as 4%. Therefore, the idea of tightly connected rectangular antennas is an intriguing combination and is still a challenging open problem for future CubeSat deployment.

IV. Based on [20], It is conceivable to show that adopting the fractal approach in teaching aspiring natural science teachers at higher educational institutions is practical. Additionally, is applying tried-and-true fractal method components to science instruction at higher education institutions feasible in the real world? The problem is still unsolvable.

V. Based on [21], some potential fractal applications to advance ChatGPT were spotlighted. This would pose a challenging open problem regarding Fractal ChatGPT's educational advancement. Would this be possible in a real-life setting? This is still open.

VI. In [22], Ismail's fourth entropy, namely $\left(\mathrm{H}_{\mathrm{IV}}^{(q,a_1,a_2,\dots,a_k)}\right)$, a novel generalization to Shannonian entropy with a visionary link to both long- and short-range interactions, (LRIs), (SRIs) respectively, was introduced. The fractal dimension of $\mathrm{H}_{\mathrm{IV}}^{(q,a_1,a_2,\dots,a_k)}$ was determined [22]. Following this pathway, can we design a revolutionary fractal geometric algorithm to advance CubSats? This is a very sophisticated open problem.

Finding answers to the difficult open issues and thoroughly investigating further extensions of Fractal Dimension Theory to additional multidisciplinary areas of human knowledge comprise the next research phase.

Author Contributions

Ismail A. Mageed was responsible for the overall conception and design of the study, the collection and analysis of data, and the drafting and revision of the manuscript. The author also provided critical insights and made substantial contributions to the interpretation of the results.

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Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this article.

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